John G. Power
Feb 5, 2003
Wakefield Note 213

A Method to Propagate Beams of Unequal Charges through the same Lattice

Abstract: In the Enhanced Transformer Ratio experiment, a train of 4 electron bunches, each with a different charge must be transported from the AWA drive gun at the beginning of the beamline, to the dielectric structure at the end of the beamline. This is a difficult problem since each beam will have a different space-charge defocusing force and a different emittance. Thus, if the magnetic lattice is optimized to transport one beam, this means that the other beams will be either over or under focused. In this Wakefield Note, I propose a solution to the problem of propogating 4 electron beams of unequal charge through the identical magnetic lattice. The method is to adjust the radius of each beam until all 4 beams have the same defocusing pressure (from both emittance and space-charge). Trace3D is used to solve for the ideal radius for each given charge in a FODO channel. Plans for a more detailed analysis, using Parmela, is discussed.

I. Introduction:

The Enhanced Transformer Ratio experiment requires that a Ramped Bunch Train (RBT) of 4 electron bunches be transported down the AWA beamline with the optimum ratio of charges between the 4 bunches, 1^{st}: 2^{nd}:3^{rd}:4^{th}, is 1:3:5:7. The difficulty with the RBT is that bunches of unequal charges, and thus different space-charge defocusing forces, be transported through the AWA beamline. Thus, if the beamline is matched for one particular charge, it is not matched for the others.

In this note, I propose that the four different charges can be transported through the same beamline by adjusting the outer radius of the beams so that they all have the same space-charge forces. This can be done by making the laser spot size in the rf photocathode gun different for each beam as described in the reference [WF note xxx]. To demonstrate this idea, I will determine the radius required to produce this condition for the special case of a periodic FODO channel. Once it is shown that these four beams can be transported through the same FODO channel, it is suggested how to conduct a similar study for the AWA beamline for the RBT experiment.

II. The Problem: How to Transport Unequal Charges Through the same Channel?

Let's begin by demonstrating the difficulty with trying to transport unequal charges through the FODO cell shown in Figure 1. To keep the analysis simple, I am using a 'thin-lens" version of the FODO cell that is symmetric about its center, with a separation between the quads (lenses) of L and with a focusing length of f. For all of the beam quality factors I assumed parameters that are relatively close to the AWA drive beam planned for the RBT experiment, i.e. E = 15 MeV, normalized transverse emittance per nanocoulomb = 1 π mm mrad, $\Delta E/E$ = 1 %, and rms bunch length = 4mm. The last two assumptions, on energy spread and bunch length, are somewhat unrealistic since they too depend on charge, but that is a detailed that can be addressed in the future by undertaking an in-depth PARMELA study.

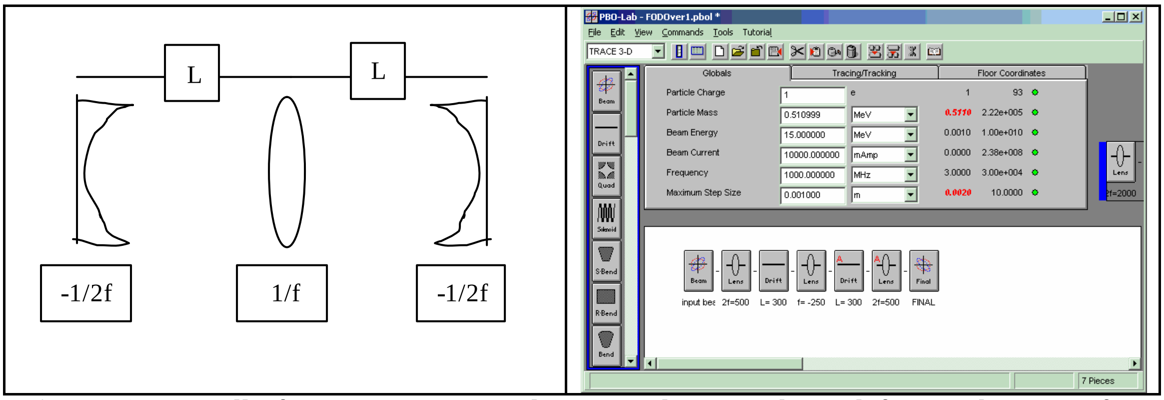


Figure 1) One cell of a symmetric, periodic FODO lattice. Shown left is a schematic of this lattice and shown right is the Trace3D graphical input file (FODO_10nC.pbol) for this lattice. For this entire paper, the FODO parameters are *L = 30 cm* and *f = 25 cm*.

The first step will be to find a *matched beam* for the Q = 10 nC case. Using the FODO cell of Figure 1, we set the input beam charge to Q=10nC, normalized$\varepsilon_n_{x,y} = 10\,\pi$ mm mrad, and use the matching feature of Trace-3D. The matched condition for this case is shown in the Trace3D output of Figure 2. The phase space plots in this figure show that beam is indeed mathched, i.e. $\alpha_{x,in} = \alpha_{x,out} = 0.288$ mrad, $\beta_{x,in} = \beta_{x,out} = 3.39$ m, $\alpha_{y,in} = \alpha_{y,out} = 0.030$ mrad, and $\beta_{y,in} = \beta_{y,out} = 0.972$ m. At the bottom of this figure we see the horizontal and vertical rms envelopes. The semi-axis parameters at the beginning and end of the channel are $\sigma_x = 1.044$ mm and $\sigma_y = 0.574$ mm.

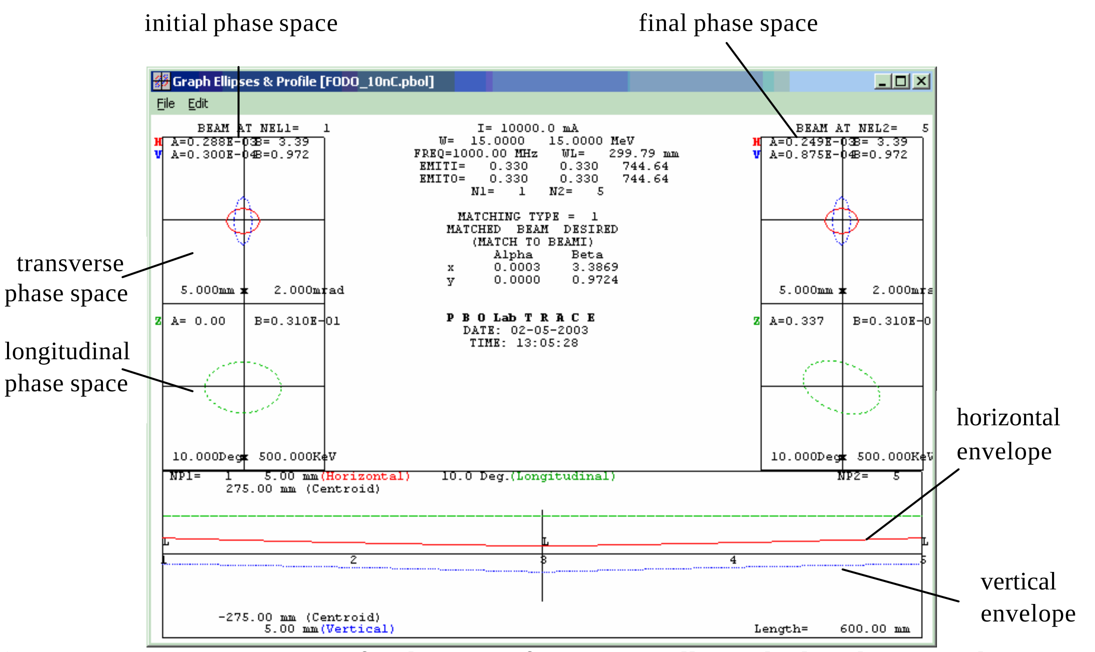


Figure 2) TRACE-3D output for the case of a FODO cell matched to the 10 nC beam. In the figure we see that the initial and final transverse phase space is the same and thus the beam is matched to the cell.

Now consider what happens when a 30 nC beam with $\varepsilon\_n_{x,y} = 30\ \pi$ mm mrad **of the same radius** as the 10 nC beam with $\varepsilon\_n_{x,y} = 10\ \pi$ mm mrad is sent through the same FODO channel. It is easily anticipated that when the higher charge beam is sent into the FODO channel of Figure 2, that its beam envelopes (horizontal and vertical) will diverge rapidly compared to the lower charge case. To quantify this statement, Trace3D is used to model the 30 nC, $\varepsilon_{x,y} = 30\ \pi$ mm mrad beam with the same radii as given above σ_x = 1.044 mm and σ_y = 0.574 mm. For the nearly upright beam used in this FODO channel the divergence is given by $\sigma' = \varepsilon/\sigma$ so $s_x' = 0.95\ mrad$ and $s_y' = 1.72\ mrad$. As is seen from Figure 3, the 30 nC beam does indeed diverge rapidly in this channel which demonstrates that a method is needed to allow both beams to pass through the channel.

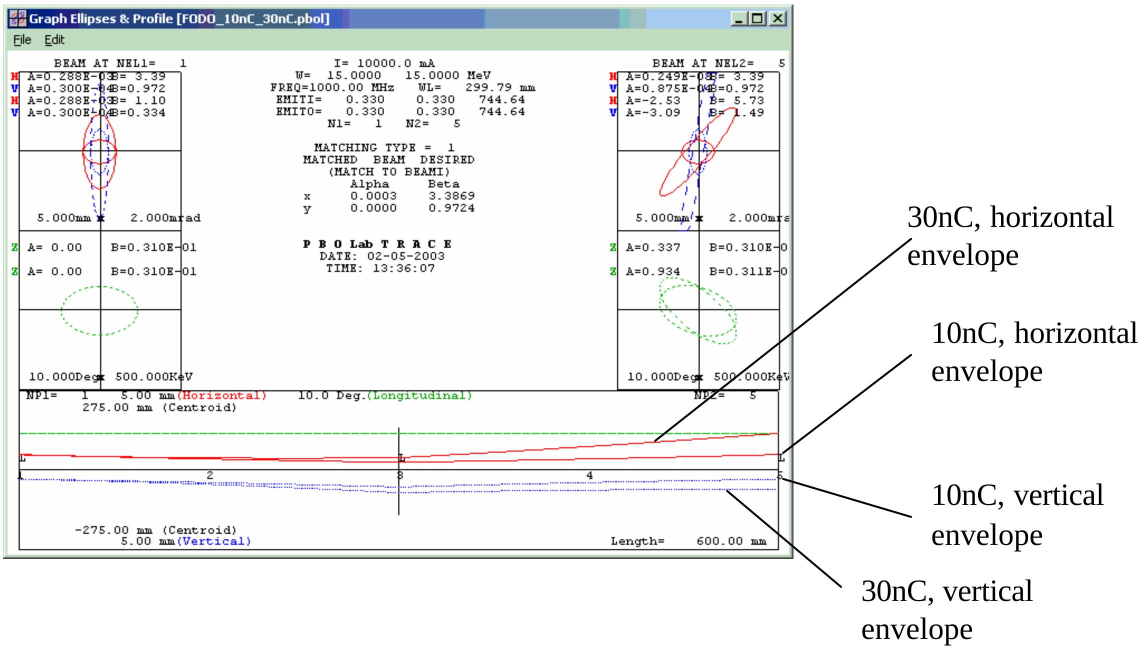


Figure 3) The beam envelopes of a 10 nC and a 30 nC beam passing through the same FODO channel of Figure 1. Note that the envelopes of the 30 nC beam diverge rapidly as the beam propogates through the channel.

III. The Solution: Vary the Radius of each Beam.

The solution to this problem can be most easily understood by a quick review of the envelope equation. The round beam envelope equation can be found in the reference [Reiser, Eqn. 4.112].

$$R''(s) + k_0^2(s)R(s) - \frac{e_r^2}{R(s)^3} - \frac{K}{R(s)} = 0 \tag{1.1}$$

where s is direction of propagation, the prime means a derivative was taken with respect to s, R is the envelope or radius of the beam, $k_0^2(s)$ is the external focusing term, e_r is the unnormalized transverse emittance, and K is the space charge parameter, or generalized perveance, which is given by

$$K = \frac{I}{I_0}\left(\frac{2}{b^3 g^3}\right) \tag{1.2}$$

where I is the peak current, $I_0 = 17000$ Amps is the *characteristic current*, and β and γ are the usual relativistic factors. As can be seen from the beam envelope equation, both the space charge and the emittance act to defocus the beam so external fosucing is used to keep the beam from diverging. If we consider the average value of the external focusing term, $\overline{k_0^2(s)} = k_0^2$, we know that we have a smooth matched beam ($\overline{R''(s)} = 0$) when the external focusing is equal to the internal beam pressure due to space charge and emittance (in an average sense) or when,

$$k_0^2 R_0 = \frac{e_r^2}{R_0^3} + \frac{K}{R_0} \tag{1.3}$$

As a first approximation, since we are in the space charge dominated regime, we will neglect the emittance term in (1.3) to see how the equilibrium beam radius should scale. In this case the equilibrium radius is found to be,

$$R_0 = \sqrt{\frac{K}{k_0^2}} \tag{1.4}$$

For a given focusing channel the external focusing term, k_0^2, which means that, in the space charge dominated rgime, we expect the equilibrium beam radius to scale like the $\sqrt{K}$. From (1.2) we know that K is proportional to the beam current and thus it is proportional to the beam charge Q or,

$$R_0 \propto \sqrt{Q} \tag{1.5}$$

This means that if the matched beam radius was $R_0 = 1\ mm$ at a charge of 10 nC for a given focusing channel, then for a charge of $Q = 30\ nC$ we would expect the new matched beam radius to be the $R_0 = (1\ mm) * \sqrt{30/10} = 1.73\ mm$.

Using the scaling law of (1.5) we can now predict how the ratio of the beam radius should vary along the RBT in order to match all the beams into the focusing channel. Given a ramped charge distribution of Q[nC] ={10,30,50,70} the 'space-charge' dominated scaling predicts R_0 [mm] = {1, sqrt(3), sqrt(5), Sqrt(7)} = {1, 1.73, 2.24, 2.65}. As we will see in the next section, this turns out to be a fairly good estimate. In order to check the accuracy of this assumption, we must now include the emittance term as well, this is the subject of the next section.

IV. The Optimum Radius for the Q=10-30-50-70 RBT.

In this section, Trace-3D is used to determine the ideal beam radii for the case of the Enhanced Transformer Ratio experiment with the RBT = 10nC, 30nC, 50nC, and 70 nC. Note that Trace3D includes both the emittance and space charge terms of (1.3).

After listing the beam parameters used in the simulation, I begin by matching the 10nC beam into the FODO cell of Figure 1 and repeat this for the other 3 cases.

A. For All Cases

This list of beam parameters is common to all 4 cases,

E=15MeV
$\Delta E/E$ = 1 %
rms bunch length = 4 mm

In the next 4 subsections (B-E) I give the beam parameters used in the simulation and the graphical Trace3D output to give the reader a quick, qualitative view of what is happening. Note that the scales used in the following 4 plots are identical to allow for easy comparison. I did not put all 4 cases on a single plot since it becomes too difficult to read. After presenting the graphical output in subsections B-E, I summarize my findings and give numerical output in the two tables of subsection F.

B. The 10 nC beam.

In this subsection I find a matched beam solution into the FODO cell of. The beam parameters used for this case are:

Q=10nC
e_rms_n = 10 π mm mrad

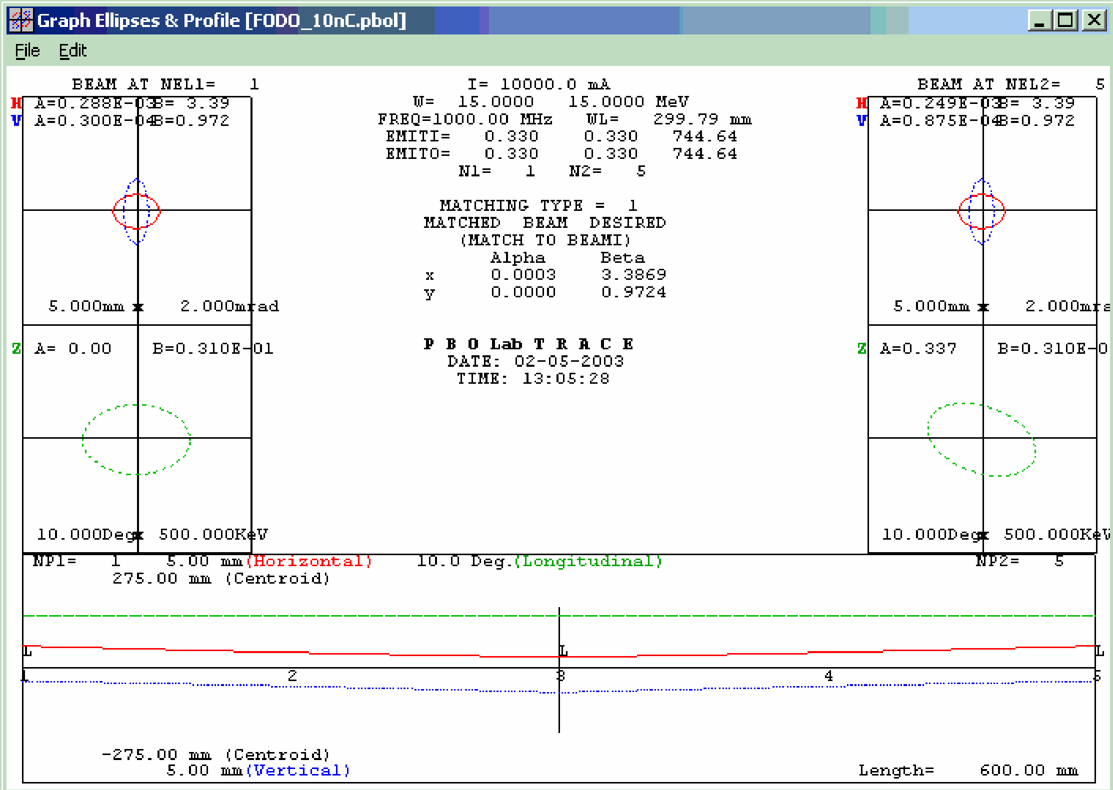


Figure 4) The Trace3D ouput file for the Q=10 nC case. See Figure 2 for the interpretation of the Trace3D output.

C. The 30 nC beam.

In this subsection I find a matched beam solution into the FODO cell of Figure 1. The beam parameters used for this are:

Q=30nC
e_rms_n = 30 π mm mrad

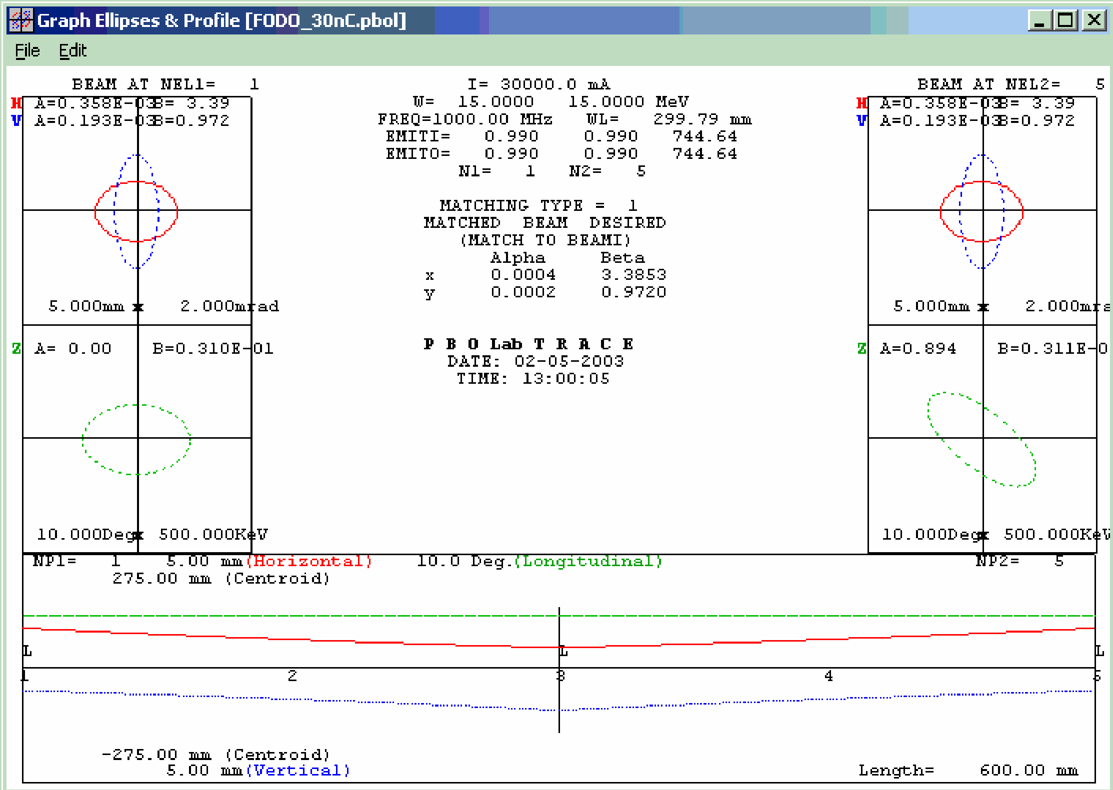
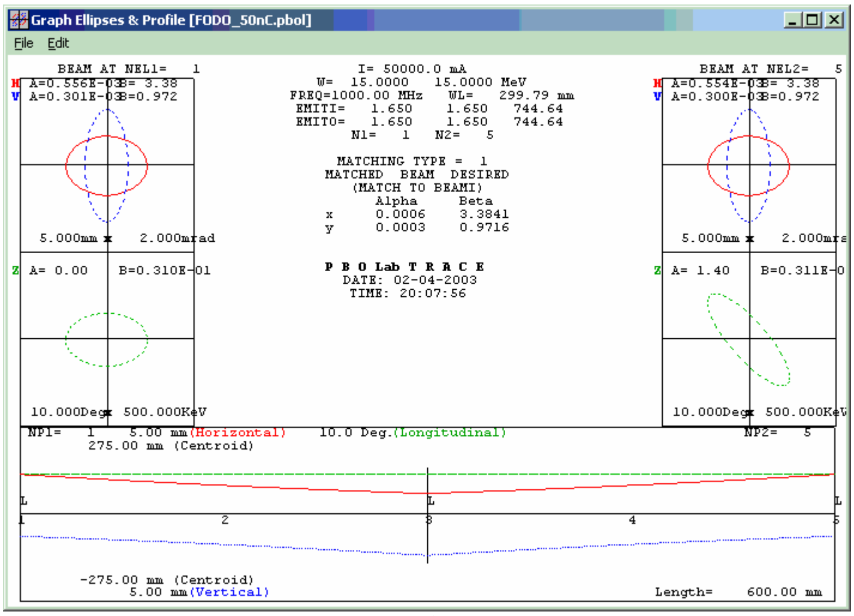


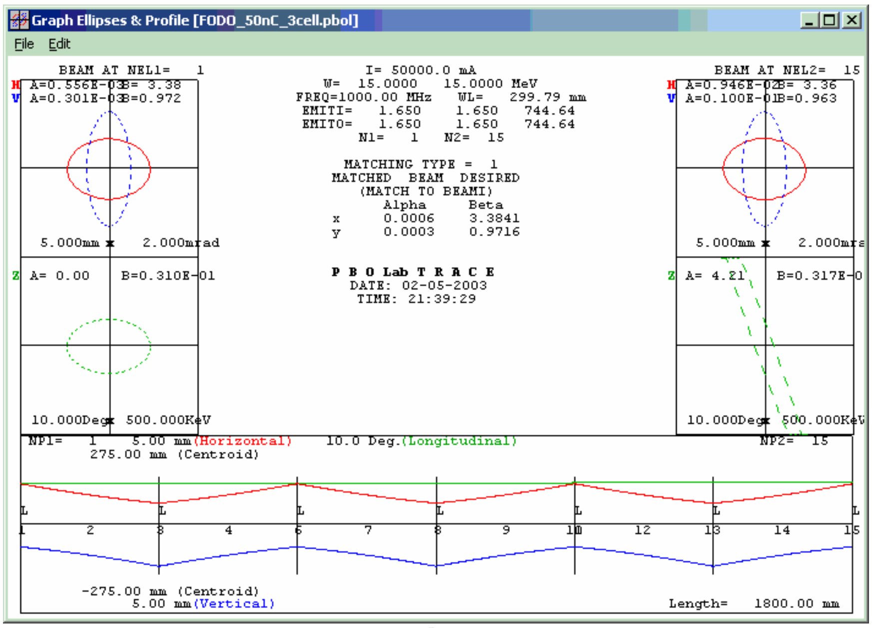
Figure 5) The Trace3D ouput file for the Q=30 nC case. See Figure 2 for the interpretation of the Trace3D output.

D. The 50 nC beam.

In this subsection I find a matched beam solution into the FODO cell of Figure 1. The beam parameters used for this are: Q=50nC & e_rms_n = 50 π mm mrad



(a)



(b)

Figure 6) The Trace3D ouput file for the Q=50 nC case. See Figure 2 for the interpretation of the Trace3D output. (a) Propagation through 1 FODO cell and (b) propagation through 3 FODO cells.

E. The 70 nC beam.

In this subsection I find a matched beam solution into the FODO cell of Figure 1. The beam parameters used for this are:

Q=50nC
e_rms_n = 50 π mm mrad

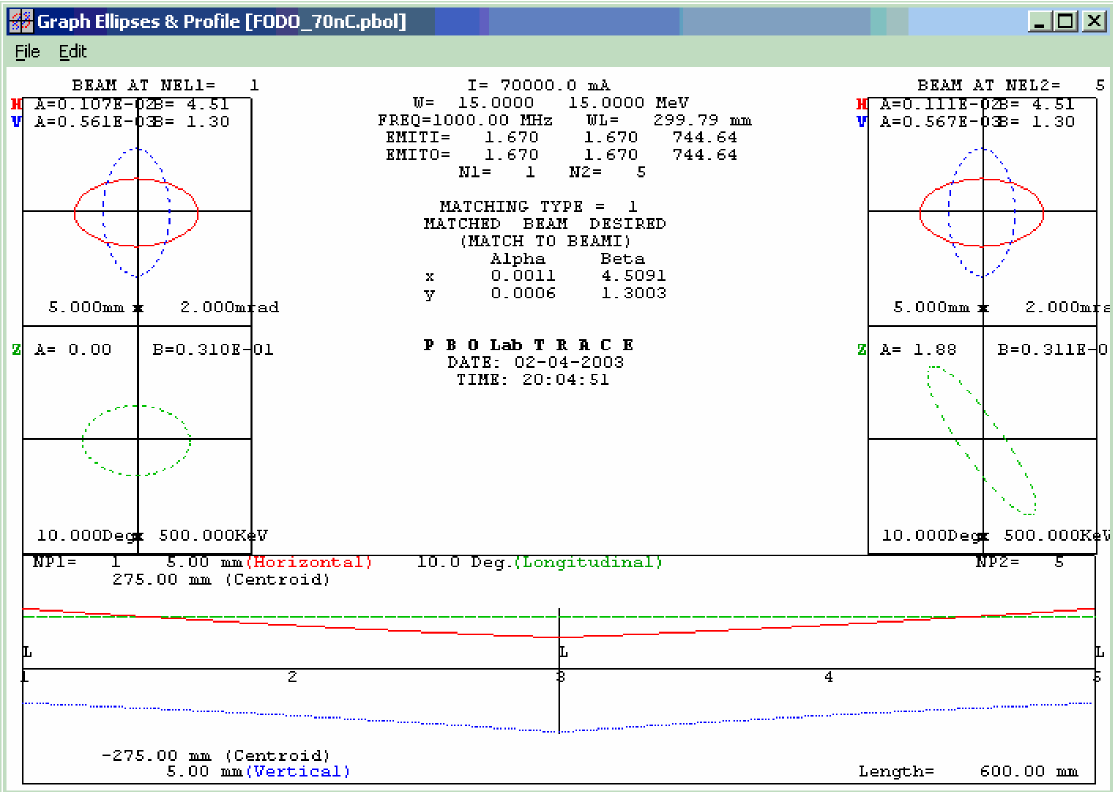


Figure 7) The Trace3D ouput file for the Q=70 nC case. See Figure 2 for the interpretation of the Trace3D output.

F. Numerical Output.

I now give the quantitative output from plots shown in the last 4 subsections. In addition to the output for the RBT = 10nC, 30nC, 50nC, and 70 nC, I have added an entry for Q = 1nC for reference.

Table 1 lists the Twiss parameters at the beginning (and end) of the FODO channel of Figure 1 for each of the cases. Note that the beta functions are the same for each of the cases, but the alpha function is slightly different from case to case.

Table 1) Trace3D derived Twiss Parameters for the 4 matched beams of the RBT.

Q (nC)	$\varepsilon_{x,y}$ (pi mm mrad)	α_x (mrad)	β_x (mm/mrad)	α_y (mrad)	β_y (mm/mrad)	ε_z(pi deg keV)	α_z	β_z (deg/keV)
1	0.033	0	3.387	0	0.973	744	0	0.0310
10	0.33	0.288	3.387	0.030	0.972	744	0	0. 0310
30	0.99	0.358	3.385	0.193	0.972	744	0	0.0310
50	1.65	0.556	3.384	0.301	0.971	744	0	0.0310
70	2.3	0.785	3.395	0.399	0.975	744	0	0.0310

For our purposes, Table 2 is the more interesting one since it tells us what the transverse size of the beam should be in order to match into the channel. Table 2 lists the Semi-axis parameters of the uniform equivalent beam at the beginning (and end) of the FODO channel of Figure 1 for each of the cases. The important columns for us to notice are the ones for σ_x and σ_y.

Table 2) Trace3D derived Semi-Axes Parameters for the 4 matched beams of the RBT.

Q (nC)	$\varepsilon_{x,y}$ (pi mm mrad)	σ_x (mm)	$\sigma_{x'}$ (mrad)	σ_y (mm)	$\sigma_{y'}$ (mrad)	σ_z (mm)	dp/p_0 (%)
1	0.033	0.334	0.099	0.179	0.184	4	1
10	0.33	1.044	0.329	0.574	0.581	4	1
30	0.99	1.807	0.559	0.995	1.001	4	1
50	1.65	2.363	0.698	1.266	1.303	4	1
70	2.3	2.795	0.823	1.497	1.536	4	1

We can now compare the Trace3D optimized transverse beam sizes to the scaling law found in equation (1.5). If we take σ_x = 1.044 mm as the starting point, Eqn. (1.5) predicts σ_x [mm] = {1.044, 1.808, 2.334, 2.762} which is very close to the Trace3D output listed in Table 1. Repeating the calculation for the *y* dimension, we take σ_y = 0.574 mm as the starting point and using Eqn. (1.5) predicts σ_y [mm] = {0.574, 0.994, 1.284, 1.519} which, again, is in excellent agreement with the Trace3D output of Table 2. The fact that estimates derived with Eqn. (1.5) are so close to the Trace3D is due to the fact that the beams are all in the space-charge dominated regime.

V. Implementation.

Although I have shown that the RBT can be propagated through the same lattice if the radii of the beams are individually adjusted, I have not discussed how to generate these beams. Since this is really a separate topic, I plan to write another wakefield note that describes how to do this, but I will briefly explain it here.

The way to generate 4 electron pulses of different charge and different radius in a photocathode gun, is to generate 4 laser pulses with different energy and different radius. In a previous wakefield note [jp] I described a *modified laser multisplitter* that could generate laser pulses of different energy, but the same radius, by replacing the 50/50 beam splitters of the *standard multisplitter* [conde] with non-50/50 splitters. Now, in order to generate different laser pulses with different radius, I suggest that we: (1) put an expanding telescope in the two delay legs of magnification M1 and M2; and (2) put an aperature after the multisplitter to set radius of the last laser pulse. With this scheme, the first laser pulse will have a radius equal to the laser pulse entering the multisplitter, say r1=1.044 mm, the second pulse will have radius r2=M1*r1, the third will have r3=M2*r1, and the fourth will have r4=M1*M2*r1. If we choose M1=s qrt(3) and M2=sqrt(5) then we have r1=1.044 mm, r2=1.808 mm, r3=2.334 mm, and r4= 4.04 mm. Comparing these values to values of the mathched radii listed in Table 2, we that the match is perfect except that r4 is too big. This, however, can be solved by placing an aperature with radius = 2.795 mm after the multisplitter. In this way the first three beams pass though undisturbed, but the 4 laser pulse gets clipped.

I note here two concerns that will need to be investigated further: (1) diffraction effects due to the aperature will need to be studied; and (2) the energy ratio in laser pulse number four will need to be adjusted so that, after it is clipped, it has the right energy needed to for the photoelectron beam.

VI. Conclusion

I have shown that electron bunches of different charges can be propagated through the same magnetic lattice by adjusting their radii. In the case of the Enhanced Transformer Ratio experiment, the radii in the RBT scale like the square root of the charge since we are in the space-charge dominated regime. In a future wakefield note a scheme for producing these bunches will be described. The next phase in the development for this technique will be to run detailed Parmela s imulations to determine the actual beam radii for the RBT.